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INFLUENCE OF ELLIPTICAL DISTRIBUTION OF LIFT ON STRENGTH OF AIRPLANE WINGS.

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INFLUENCE OF ELLIPTICAL DISTRIBUTION OF LIFT ON STRENGTH OF AIRPLANE WINGS.*

By Col. Dorand.

Hitherto it has been generally assumed, in calculating the cell of an airplane, that the forces withstood by the latter were distributed uniformly throughout the whole length of the wing. In reality this is not the case and German engineers in particular are now assuming an elliptical distribution of the forces.

The latter hypothesis has made it possible to carry out a certain number of calculations which have been verified by experiment. Consequently we may assume it to be more reasonable than the former hypothesis.

For the section A_1 (Fig. 1), located at a distance x_1 from the center of a wing with a span of $2l$, the bending moment will be M_1 according to the former hypothesis and M_2 according to the latter. We can easily calculate these two moments, if we know the total force P withstood by the wing.

It is interesting to calculate the ratio $\frac{M_1}{M_2}$ and consequently $\frac{M_1 - M_2}{M_2}$ which characterizes the excess strength of an airplane on the hypothesis of uniform distribution. This is true, since with equality of the moments of inertia, the coefficient of safety is inversely proportional to the bending moment.

* From Premier Congrés International de la Navigation Aérienne, Paris, November, 1921, Vol. II, pp. 44-46.

1. Under the hypothesis of the uniform distribution, we have:

$$M_1 = \frac{P}{2l} \times \frac{(l - x_1)^2}{2}$$

or with reference to the equation $z_1 = \frac{x_1}{l}$ independent of the span

$$M_1 = \frac{Pl}{4}(1 - z_1)^2 .$$

2. Under the hypothesis of the elliptical distribution, we have y_1 as the lift per unit length at the center of the wing (Fig. 2) and y at a point A at a distance x from the center x_1, y and y_1 are combined by the equation of the ellipse

$$y = \frac{y_1}{l}\left(l^2 - x^2\right)^{1/2} .$$

Furthermore, P is equal to half the area of the ellipse constructed with y_1 and l as the half-axes. We then have

$$P = \frac{3.1416 \times l y_1}{2} .$$

Let us take a point A (Fig. 2). At this point the force will be $y\,d\,x$ and the moment $d\,M_2$ of this force with reference to A_1, located at a distance x_1 from the center of the span, will be

$$d\,M_2 = (x - x_1)\,y\,d\,x = (x - x_1)\frac{y_1}{l}\left(l^2 - x^2\right)^{1/2} dx.$$

The total moment M_2 with reference to A_1 will then be

$$M_2 = \int_{x_1}^{l} (x - x_1)\frac{y_1}{l}\left(l^2 - x^2\right)^{1/2} dx$$

or on putting $z_1 = \frac{x_1}{l}$

$$M_2 = \frac{2\,Pl}{3.1416}\left[\frac{(1 - z_1^2)^{3/2}}{3} + \frac{z_1^2(1 - z_1^2)^{1/2}}{2} + \frac{z_1}{2}\right.$$
$$\left.\text{arc sin } z_1 - \frac{3.1416\ z_1}{4}\right]$$

On designating the parenthetical term by Σ, we have

$$M_2 = \frac{2\,P}{3.1416} \times \Sigma$$

The ratio $\frac{M_1}{M_2}$ will then be $\frac{M_1}{M_2} = \frac{3.1416\ (1 - z_1)^2}{8\ \Sigma}$

independent of the span and of the total force P.

If we calculate the values of the ratios $\frac{M_1}{M_2}$ and $\frac{M_1 - M_2}{M_2}$ in terms of $z_1 = \frac{x_1}{l_1}$, we have

1 $z = \frac{x_1}{l_1}$	2 $\frac{M_1}{M_2}$	4	3 Excess strength $\frac{M_1 - M_2}{M_2}$
0	8	Center of airplane	18%
0.2	1.28		28%
0.4	1,46		46%
0.6	1.75		75%
0.8	2.6		160%
1		Extreme edge of wing	

If, as we must assume, the distribution of the forces withstood by the wings approaches the elliptical form, the airplanes calculated on the assumption of uniform distribution would not be homogeneous, as shown by the figures of the above table.

In fact, their excess strength, which is considerable at the ends of the wings, would continue to decrease toward the center of the span, where this excess would still be 18% of the total strength At the middle of the half-span the excess strength would be about 50%.

The attention of constructors should be directed especially to the point just mentioned, since the foregoing considerations lead us to anticipate considerable reductions in the weight of airplane wings, which would greatly affect their manner of construction.

The static tests would have to correspond to any new method of construction, consequent on the foregoing remarks, with allowance for the elliptical distribution.

Of course the hypothesis of the elliptical distribution should not be accepted without verification, but to follow old erroneous methods, without investigating the possibility of a great improvement in construction, would be contrary to the idea of progress which has always animated us in our work.

Translated by the National Advisory Committee for Aeronautics.

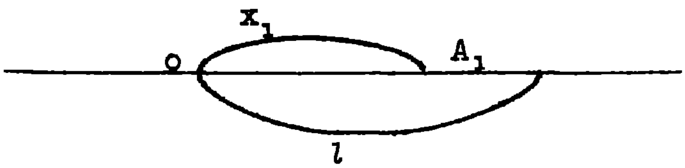

Fig. 1.

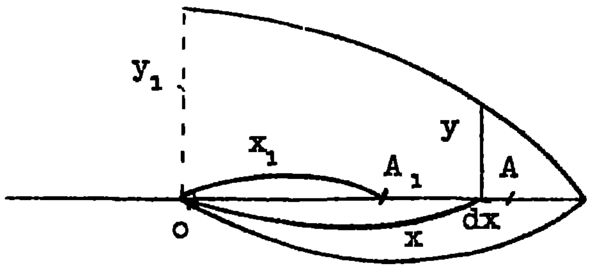

Fig. 2.

The difference is that whereas in the case of the Maurice Farman the machine was doing these slow glides with a loading of 2 lbs. to the square foot and whereas the B.Es were doing it with a loading of about 4 lbs. to the square foot, the Handley Page did it at 8 lbs. to the square foot. When one gets beyond such weight it is possible by eliminating official gadgets to produce a machine which comes very near carrying a paying commercial load without the aid of subsidies. That is where the real advance has been made. (C.G.G.)